Mathematical Design Models for Self-Organizing Systems*

Patrick Wüchner, Christopher Auer, Richard Holzer, and Hermann de Meer

Faculty of Informatics and Mathematics,
University of Passau, Innstraße 43,
94032 Passau, Germany
{wuechner,auer,holzer,demeer}@fim.uni-passau.de

Self-organization (SO) appears in many different fields: in computer science (e.g., sensor networks), biology (e.g., colony of ants), physics (e.g., elementary magnets), and many more. During the last years, engineering sciences have been attracted by the phenomenon of SO to overcome inevitable issues, for example, scalability issues that arise when engineering and managing large-scale systems consisting of numerous interacting entities. However, prior to engineering self-organizing systems (SOSs), a common terminology is needed to characterize and define SO and SO-related concepts.

An engineer's primary goal is to create systems which fulfill a predefined task. This is captured by the concept of *organization*: We call a system organized if it is able to promote a purposeful function.

Furthermore, the "self" in SO implies that the system is able to fulfill its task without being explicitly controlled by an external entity. The concept of *autonomy* captures this property.

Another important property of SOSs is their robustness with respect to perturbations in their environment, i.e., the SOSs dynamically adapt to changing external situations in order to guarantee that their organization stays intact. This is what is commonly defined to be *adaptivity*.

The most intriguing property of SOSs is that a novel global behavior at the *macro level* emerges from the interactions of the numerous entities that constitute the system's *micro level*. However, the macro-level behavior is not displayed in the micro-level entities, and hence, cannot be seen by studying a single component independently from the others. This appealing phenomenon is called *emergence*.

Having identified these key properties of an SOS, we give the following definition of SO: A system is *self-organizing* if its is autonomous, adaptive, and its organization is an emergent property.

Another frequently appearing phenomenon in SOSs is *feedback* that refers to a part of the output, of one or several entities, which is returned to the same entities as input (see [1]).

* This research is partially supported by the AutoI project (STREP, FP7 Call1, ICT-2007-1-216404) and by the Networks of Excellence EuroFGI (IST 028022) and EuroNF (IST 216366).

We distinguish three different kinds of feedback loops:

– Environmental feedback loop: The SOS interacts with the environment. This might change the environment's state which again influences the SOS.
– Cross-layer feedback loop: The interactions of the entities on micro level lead to an emergent pattern on macro level. This pattern has direct influence on the behavior of the micro-level entities.
– Engineer feedback loop: The emerging pattern on macro level can be inspected by an engineer able to evaluate the performance and modify the micro-level entities of the SOS.

Unfortunately, these informal definitions lack mathematical rigor. Formal mathematical system models can help to analyze and compare systems with respect to the self-organizing properties and allow to formally define and objectively evaluate properties like emergence or autonomy (see [2]).

There are two main approaches towards the mathematical modeling of SOSs: Micro-level modeling and macro-level modeling. When applying micro-level modeling, each entity's own state space is considered. Hence, the state space of the whole system can be seen as the Cartesian product of the state spaces of all entities. The behavior of the entities can be modeled by (deterministic or stochastic) automatons: Each entity receives input from other entities, changes its internal state, and sends some output to other entities. The micro-level modeling is more exact, but does not scale since it suffers from state-space explosion.

When modeling SOSs on macro level, equivalent micro-level states are aggregated into macro-level states to reduce the size of the state space. In many situations, there is no need to describe the behavior of each micro-level entity, but only the change of the aggregated global state has to be considered. However, this in general leads to a lower granularity in the models. A further disadvantage of macro-level modeling is that it might be hard to map the conclusions from macro-level analysis to micro-level design directives (see [3]).

The main objective of our research is to aid the design, engineering, and optimization of large-scale technical SOSs by proposing a common terminology and by providing formal descriptions for SO and related concepts, like emergence and evolution.

References

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